

Department of Mathematics, University of Delhi

UNIVERSITY OF DELHI
DEPARTMENT OF MATHEMATICS
GENERIC ELECTIVE (GE) Courses for Honours Courses
(For students other than B.Sc. (Hons.) Mathematics)

Learning Outcomes based Curriculum Framework (LOCF)
2019



**GENERIC ELECTIVE (GE) COURSES OFFERED TO
B.Sc. (Hons.) / B.A. (Hons.) / B.Com (Hons.)
(Other than B.Sc. (Hons.) Mathematics)**

| Sem-ester | Core Course (14) | Ability Enhancement Compulsory Course (AECC)(2) | Skill Enhancement Course (SEC) (2) | Discipline Specific Elective (DSE)(4) | Generic Elective (GE) (4) Credits: 6 each |
|-----------|------------------|---|------------------------------------|---------------------------------------|---|
| I | | | | | GE-1 Calculus OR Analytic Geometry and Theory of Equations |
| II | | | | | GE-2 Linear Algebra OR Discrete Mathematics |
| III | | | | | GE-3 Differential Equations (with Practicals) OR Linear Programming and Game Theory |
| IV | | | | | GE-4 Numerical Methods (with Practicals) OR Elements of Analysis |
| V | | | | | |
| VI | | | | | |

Semester-I Generic Elective (GE) Course -Mathematics

Any *one* of the following:

GE-1: Calculus

GE-1: Analytic Geometry and Theory of Equations

GE-1: Calculus

Total Marks: 100 (Theory: 75, Internal Assessment: 25)

Workload: 5 Lectures, 1 Tutorial (per week) Credits: 6 (5+1)

Duration: 14 Weeks (70 Hrs.) Examination: 3 Hrs.

Course Objectives: The main aim of this course is to learn about applications of derivatives for sketching of curves and conics and applications of definite integrals for calculating volumes of solids of revolution, length of plane curves and surface areas of revolution. Various notions related to vector-valued functions and functions of several variables are discussed in this course.

Course Learning Outcomes: This course will enable the students to:

- i) Sketch the curves in Cartesian and polar coordinates as well as learn techniques of sketching the conics.
- ii) Visualize three dimensional figures and calculate their volumes and surface areas.
- iii) Understand limits, continuity and derivatives of functions of several variable and vector-valued functions.

Unit 1: Applications of Derivatives and Limits

The first derivative test, Concavity and inflection points, Second derivative test, Curve sketching using first and second derivative test; Limits at infinity, Horizontal asymptotes, Vertical asymptotes, Graphs with asymptotes; L'Hôpital's rule.

Unit 2: Applications of Definite Integrals

Volumes by slicing, Volumes of solids of revolution by the disk method, Volumes of solids of revolution by the washer method, Volume by cylindrical shells, Length of plane curves, Arc length of parametric curve, Area of surface of revolution.

Unit 3: Conics, Vector-Valued Functions and Partial Derivatives

Techniques of sketching conics, Reflection properties of conics; Polar coordinates, graphing in polar coordinates; Vector-valued functions: Limits, Continuity, Derivatives, Integrals, Arc length, Unit tangent vector, Curvature, Unit normal vector; Functions of several variables: Graphs and level curves, Limits and continuity, Partial derivatives and differentiability, The chain rule, Directional derivatives and gradient vectors, Tangent plane and normal line, Extreme values and saddle points.

References:

1. Anton, Howard, Bivens, Irl, & Davis, Stephen (2013). *Calculus* (10th ed.). John Wiley & Sons Singapore Pvt. Ltd. Reprint (2016) by Wiley India Pvt. Ltd. Delhi.
2. Strauss, M. J., Bradley, G. L., & Smith, K. J. (2007). *Calculus* (3rd ed.). Dorling Kindersley (India) Pvt. Ltd. (Pearson Education). Delhi. Sixth impression 2011.

Additional Reading:

- i. Thomas, Jr. George B., Weir, Maurice D., & Hass, Joel (2014). *Thomas' Calculus* (13th ed.). Pearson Education, Delhi. Indian Reprint 2017.

Teaching Plan (GE-1: Calculus):

Weeks 1 and 2: The first derivative test, Concavity and inflection points, Second derivative test, Curve sketching using first and second derivative test.

[2] Chapter 4 (Section 4.3).

Weeks 3 and 4: Limits at infinity, Horizontal asymptotes, Vertical asymptotes, Graphs with asymptotes; L'Hôpital's rule.

[2] Chapter 4 (Sections 4.4, and 4.5).

[1] Chapter 3 (Section 3.3), and Chapter 6 (Section 6.5).

Weeks 5 and 6: Volumes by slicing, Volumes of solids of revolution by the disk method, Volumes of solids of revolution by the washer method, Volume by cylindrical shells.

[1] Chapter 5 (Sections 5.2, and 5.3).

Week 7: Length of plane curves, Arc length of parametric curves, Area of surface of revolution.

[1] Chapter 5 (Sections 5.4, and 5.5).

Week 8: Techniques of sketching conics, Reflection properties of conics.

[1] Chapter 10 (Section 10.4).

Week 9: Polar coordinates, Graphing in polar coordinates.

[1] Chapter 10 (Section 10.2).

Week 10: Vector-valued functions: Limit, continuity, Derivatives, Integrals, Arc length, Unit tangent vector, Curvature, Unit normal vector.

[1] Chapter 12 (Sections 12.1 to 12.5).

Weeks 11 and 12: Functions of several variables: Graphs, Level curves, Limits and continuity, Partial derivatives and differentiability.

[1] Chapter 13 (Section 13.1 to 13.4).

Week 13: Functions of several variables: The chain rule, Directional derivatives and gradient vectors.

[1] Chapter 13 (Sections 13.5, and 13.6).

Week 14: Functions of several variables: Tangent plane and normal line, Extreme values and saddle points.

[1] Chapter 13 (Sections 13.7, and 13.8).

Facilitating the Achievement of Course Learning Outcomes

| Unit No. | Course Learning Outcomes | Teaching and Learning Activity | Assessment Tasks |
|----------|---|--|---|
| 1. | Sketch the curves in Cartesian and polar coordinates as well as learn techniques of sketching the conics. | (i) Each topic to be explained with examples. | <ul style="list-style-type: none"> • Student presentations. • Participation in discussions. • Assignments and class tests. • Mid-term examinations. • End-term examinations. |
| 2. | Visualize three dimensional figures and calculate their volumes and surface areas. | (ii) Students to be involved in discussions and encouraged to ask questions. | |
| 3. | Understand limits, continuity and derivatives of functions of several variable and vector-valued functions. | (iii) Students to be given homework/ assignments. (iv) Students to be encouraged to give short presentations. | |

Keywords: Concavity, Asymptotes, Curve sketching, L'Hôpital's rule, Volumes of solids of revolution, Sketching of conics, Vector-valued functions, Functions of several variables.

GE-1: Analytic Geometry and Theory of Equations

Total Marks: 100 (Theory: 75, Internal Assessment: 25)

Workload: 5 Lectures, 1 Tutorial (per week) Credits: 6 (5+1)

Duration: 14 Weeks (70 Hrs.) Examination: 3 Hrs.

Course Objectives: The goal of this paper is to acquaint students with certain ideas about conic sections, vectors in coordinate system and general properties of roots of polynomial equations with some applications.

Course Learning Outcomes: After completion of this paper, the students will be able to:

- i) Classify and sketch conics four different types of conic sections – the circle, the ellipse, the hyperbola and the parabola – in Cartesian and polar coordinates.
- ii) Visualize three dimensional objects – spheres and cylinders – using vectors.
- iii) Understand the properties of roots of polynomial equations.

Unit 1: Conic Sections, Parametrized Curves, and Polar Coordinates

Conic sections and quadratic equations: Circle, Parabola, Ellipse, and hyperbola; Techniques for sketching: Parabola, Ellipse, and Hyperbola; Reflection properties of parabola, ellipse, and hyperbola, Classifying conic sections by eccentricity, Classification of quadratic equations representing lines, parabola, ellipse, and hyperbola; Parameterization of plane curves, Conic sections in polar coordinates and their sketching.

Unit 2: Three-Dimensional Space: Vectors

Rectangular coordinates in 3-space, Spheres and cylindrical surfaces, Vectors viewed geometrically, Vectors in coordinate systems, Vectors determined by length and angle, Dot product, Cross product and their geometrical properties, Parametric equations of lines in 2-space and 3-space.

Unit 3: Theory of Equations

General properties of polynomials and equations, Descartes' rule of signs for positive and negative roots, Relation between the roots and the coefficients of equations, Applications, Depression of an equation when a relation exists between two of its roots, Symmetric functions of the roots and its applications, Transformation of equations (multiplication, reciprocal, increase/diminish in the roots by a given quantity), Removal of terms; Graphical representation of derived function, Rolle's theorem, Multiple roots of the equation.

References:

1. Anton, Howard, Bivens, Irl, & Davis, Stephen (2013). *Calculus* (10th ed.). John Wiley & Sons Singapore Pvt. Ltd. Reprint (2016) by Wiley India Pvt. Ltd. Delhi.
2. Burnside, W.S., & Panton, A.W. (1979), *The Theory of Equations*, Vol. 1. Eleventh Edition, Fourth Indian Reprint. S. Chand & Co. New Delhi.
3. Thomas, Jr. George B., Weir, Maurice D., & Hass, Joel (2014). *Thomas' Calculus* (13th ed.). Pearson Education, Delhi. Indian Reprint 2017.

Additional Readings:

- i. Dickson, Leonard Eugene (2009). *First Course in the Theory of Equations*. The Project Gutenberg EBook (<http://www.gutenberg.org/ebooks/29785>).

- ii. Ferrar, W. L. (1956). *Higher Algebra*. Oxford University Press.

Teaching Plan (GE-I: Analytical Geometry and Theory of Equations):

Weeks 1 and 2: Conic sections and quadratic equations: circle, parabola, ellipse, and hyperbola; Techniques for sketching: parabola, ellipse, and hyperbola; Reflection properties of parabola, ellipse, and hyperbola.

[3] Chapter 11 (Section 11.6).

[1] Chapter 10 (Section 10.4).

Week 3: Classifying conic sections by eccentricity.

[3] Chapter 11 (Section 11.7).

Weeks 4 and 5: Classification of quadratic equations representing lines, parabola, ellipse, and hyperbola; Parameterization of plane curves, Conic sections in polar coordinates and their sketching.

[1] Chapter 10 (Section 10.2).

[3] Chapter 11 (Sections 11.1, and 11.4).

Weeks 6 and 7: Rectangular coordinates in 3-space, Spheres and cylindrical surfaces, Vectors viewed geometrically, Vectors in coordinate systems, Vectors determined by length and angle.

[1] Chapter 11 (11.1, and 11.2)

Weeks 8 and 9: Dot product, Cross product and their geometrical properties, Parametric equations of lines in 2-space and 3-space.

[1] Chapter 11 (Sections 11.3 to 11.5).

Weeks 10 and 11: General properties of polynomials and equations, Descartes' rule of signs for positive and negative roots, Relation between the roots and the coefficients of equations, Applications.

[2] Chapter 2 (Sections 12 to 22), Chapter 3 (Sections 23 and 24).

Weeks 12 and 13: Depression of an equation when a relation exists between two of its roots, Symmetric functions of the roots and its applications, Transformation of equations (multiplication, reciprocal, increase/diminish in the roots by a given quantity), Removal of terms.

[2] Chapter 3 (Sections 25 to 28), Chapter 4 (Sections 29 to 34).

Week 14: Graphical representation of derived function, Rolle's theorem, Multiple roots of the equation.

[2] Chapter 7 (Sections 69, 71, 73 and 74).

[1] Chapter 3 (Section 3.8.1).

Facilitating the Achievement of Course Learning Outcomes

| Unit No. | Course Learning Outcomes | Teaching and Learning Activity | Assessment Tasks |
|----------|---|---|---|
| 1. | Classify and sketch conics four different types of conic sections – the circle, the ellipse, the hyperbola and the parabola – in Cartesian and polar coordinates. | (i) Each topic to be explained with examples. (ii) Students to be involved in discussions and encouraged to ask questions. | <ul style="list-style-type: none"> • Student presentations. • Participation in discussions. • Assignments and class tests. • Mid-term examinations. • End-term examinations. |
| 2. | Visualize three-dimensional objects – spheres and cylinders – using vectors. | (iii) Students to be given homework/assignments. (iv) Students to be encouraged to give short presentations. | |
| 3. | Understand the properties of roots of polynomial equations. | | |

Keywords: Circle, Parabola, Ellipse, Hyperbola, Spheres, Cylindrical surfaces, Vectors, Roots of equations, Coefficients of equations.

Semester-II Generic Elective (GE) Course -Mathematics

Any one of the following:

GE-2: Linear Algebra

GE-2: Discrete Mathematics

GE-2: Linear Algebra

Total Marks: 100 (Theory: 75, Internal Assessment: 25)

Workload: 5 Lectures, 1 Tutorial (per week) **Credits:** 6 (5+1)

Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: The objective of the course is to introduce the concept of vectors in \mathbb{R}^n . The concepts of linear independence and dependence, rank and linear transformations has been explained through matrices. Various applications of vectors in computer graphics and movements in a plane has also been introduced.

Course Learning Outcomes: This course will enable the students to:

- i) Visualize the space \mathbb{R}^n in terms of vectors and the interrelation of vectors with matrices, and their application to computer graphics.
- ii) Familiarize with concepts in vector spaces, namely, basis, dimension and minimal spanning sets.
- iii) Learn about linear transformations, transition matrix and similarity.
- iv) Learn about orthogonality and to find approximate solution of inconsistent system of linear equations.

Unit 1: Euclidean space \mathbb{R}^n and Matrices

Fundamental operation with vectors in Euclidean space \mathbb{R}^n , Linear combination of vectors, Dot product and their properties, Cauchy–Schwarz inequality, Triangle inequality, Projection vectors, Some elementary results on vectors in \mathbb{R}^n , Matrices: Gauss–Jordan row reduction, Reduced row echelon form, Row equivalence, Rank, Linear combination of vectors, Row space, Eigenvalues, Eigenvectors, Eigenspace, Characteristic polynomials, Diagonalization of matrices; Definition and examples of vector spaces, Some elementary properties of vector spaces, Subspace, Span, Spanning set for an eigenspace, Linear independence and linear dependence of vectors, Basis and dimension of a vector space, Maximal linearly independent sets, Minimal spanning sets; Application of rank: Homogenous and non-homogenous systems of linear equations; Coordinates of a vector in ordered basis, Transition matrix.

Unit 2: Linear Transformations and Computer Graphics

Linear transformations: Definition and examples, Elementary properties, The matrix of a linear transformation, Linear operator and similarity; Application: Computer graphics, Fundamental movements in a plane, Homogenous coordinates, Composition of movements; Kernel and range of a linear transformation, Dimension theorem, One to one and onto linear transformations, Invertible linear transformations, Isomorphism, Isomorphic vector spaces (to \mathbb{R}^n).

Unit 3: Orthogonality and Least Square Solutions

Orthogonal and orthonormal vectors, Orthogonal and orthonormal bases, Orthogonal complement, Projection theorem, Orthogonal projection onto a subspace; Application: Least square solutions for inconsistent systems, Non-unique least square solutions.

References:

1. Andrilli, S., & Hecker, D. (2016). *Elementary Linear Algebra* (5th ed.). Elsevier India.
2. Kolman, Bernard, & Hill, David R. (2001). *Introductory Linear Algebra with Applications* (7th ed.). Pearson Education, Delhi. First Indian Reprint 2003.

Additional Reading:

- i. Lay, David C., Lay, Steven R., & McDonald, Judi J. (2016). *Linear Algebra and its Applications* (5th ed.). Pearson Education.

Teaching Plan (GE-2: Linear Algebra):

Week 1: Fundamental operation with vectors in Euclidean space \mathbb{R}^n , Linear combination of vectors, dot product and their properties, Cauchy–Schwarz inequality, Triangle inequality, Projection vectors.

[1] Chapter 1 (Sections 1.1 and 1.2).

Week 2: Some elementary results on vectors in \mathbb{R}^n ; Matrices: Gauss–Jordan row reduction, Reduced row echelon form, Row equivalence, Rank.

[1] Chapter 1 [Section 1.3 (Pages 34 to 44)].

[1] Chapter 2 [Sections 2.2 (up to Page 111), 2.3 (up to Page 122, Statement of Theorem 2.5)].

Week 3: Linear combination of vectors, Row space, Eigenvalues, Eigenvectors, Eigenspace, Characteristic polynomials, Diagonalization of matrices.

[1] Chapter 2 [Section 2.3 (Pages 122-132, Statements of Lemma 2.8, Theorem 2.9)], Chapter 3 (Section 3.4).

Week 4: Definition and examples of vector spaces, Some elementary properties of vector spaces.

[1] Chapter 4 (Section 4.1).

Week 5 and 6: Subspace, Span, Spanning set for an eigenspace, Linear independence and dependence, Basis and dimension of a vector space, Maximal linearly independent sets, Minimal spanning sets.

[1] Chapter 4 (Sections 4.2 to 4.5, Statements of technical Lemma 4.10 and Theorem 4.12).

Week 7: Application of rank: Homogenous and non-homogenous systems of linear equations; Coordinates of a vector in ordered basis, Transition matrix.

[2] Chapter 6 [Sections 6.6 (Pages 287 to 291), and 6.7 (Statement of Theorem 6.15 and examples)].

Week 8: Linear transformations: Definition and examples, Elementary properties.

[1] Chapter 5 (Section 5.1).

Week 9: The matrix of a linear transformation, Linear operator and similarity.

[1] Chapter 5 [Section 5.2 (Statements of Theorem 5.5 and Theorem 5.6)].

Week 10: Application: Computer graphics, Fundamental movements in a plane, Homogenous coordinates, Composition of movements.

[1] Chapter 8 (Section 8.8).

Week 11: Kernel and range of a linear transformation, Statement of the dimension theorem and examples.

[1] Chapter 5 (Sections 5.3).

Week 12: One to one and onto linear transformations, Invertible linear transformations, isomorphism, isomorphic vector spaces (to \mathbb{R}^n).

[1] Chapter 5 [Sections 5.4, 5.5 (up to Page 378, Statements of Theorem 5.15, and Theorem 5.16)]

Week 13 and 14: Orthogonal and orthonormal vectors, orthogonal and orthonormal bases, orthogonal complement, statement of the projection theorem and examples. Orthogonal projection onto a subspace. Application: Least square solutions for inconsistent systems, non-unique least square solutions.

[1] Chapter 6 [Sections 6.1 (up to Example 3, Page 416, Statement of Theorem 6.3), 6.2 (up to Page 435, and Pages 439 to 443, and Statement of Theorem 6.12)].

[1] Chapter 8 [Section 8.9 (up to Page 593, Statement of Theorem 8.13)].

Facilitating the Achievement of Course Learning Outcomes

| Unit No. | Course Learning Outcomes | Teaching and Learning Activity | Assessment Tasks |
|----------|--|--|---|
| 1. | Visualize the space \mathbb{R}^n in terms of vectors and the interrelation of vectors with matrices, and their application to computer graphics. Familiarize with concepts in vector spaces, namely, basis, dimension and minimal spanning sets. | (i) Each topic to be explained with examples. (ii) Students to be involved in discussions and encouraged to ask questions. (iii) Students to be given homework/assignments. (iv) Students to be encouraged to give short presentations. | <ul style="list-style-type: none"> • Student presentations. • Participation in discussions. • Assignments and class tests. • Mid-term examinations. • End-term examinations. |
| 2. | Learn about linear transformations, transition matrix and similarity. | | |
| 3. | Learn about orthogonality and to find approximate solution of inconsistent system of linear equations. | | |

Keywords: Cauchy–Schwarz inequality Gauss–Jordan row reduction Basis and dimension of vector spaces, matrix of linear transformations, Orthogonality, Orthonormality, Least square solutions.

GE-2: Discrete Mathematics

Total Marks: 100 (Theory: 75, Internal Assessment: 25)

Workload: 5 Lectures, 1 Tutorial (per week) **Credits:** 6 (5+1)

Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: The course introduces formal logic notation, methods of proof, mathematical induction, set theory, permutations and combinations and counting principles. One can learn the concepts of lattices and Boolean algebra in analysis of various applications.

Course Learning Outcomes: This course will enable the students to:

- i) Understand the basic principles of logic, set theory, lattices and Boolean algebra.
- ii) Understand the ideas of mathematical induction and basic counting techniques.
- iii) Proficiently construct logical arguments and rigorous proofs.

Unit 1: Logical Mathematics

Compound statements (and, or, implication, negation, contrapositive, quantifiers), Truth tables, Basic logical equivalences and its consequences, Logical arguments, Set theory, Operation on sets, Types of binary relations, Equivalence relations, Congruences and its properties, Partial and total ordering, Lattices, Properties of integers, Division algorithm, Divisibility and Euclidean algorithm, GCD, LCM, Relatively prime.

Unit 2: Applications of Numbers

Prime numbers, Statement of fundamental theorem of arithmetic, Fermat primes, Mathematical induction, Recursive relations and its solution (characteristics polynomial and generating function), Principles of counting (inclusion/exclusion, pigeon-hole), Permutation and combinations (with and without repetition).

Unit 3: Lattices and its Properties

Duality principle, Lattices as ordered sets, Lattices as algebraic structures, Sublattices, Products and homomorphisms, Distributive lattices, Boolean algebras, Boolean polynomials, Minimal forms of Boolean polynomials, Quinn–McCluskey method, Karnaugh diagrams, Switching circuits and applications of switching circuits.

References:

1. Davey, B A., & Priestley, H. A. (2002). *Introduction to Lattices and Order* (2nd ed.). Cambridge University Press. Cambridge. 7th Printing 2012.
2. Goodaire, Edgar G., & Parmenter, Michael M. (2003). *Discrete Mathematics with Graph Theory* (2nd ed.). Pearson Education (Singapore) Pte. Ltd. Indian Reprint.
3. Lidl, Rudolf & Pilz, Günter. (1998). *Applied Abstract Algebra* (2nd ed.). Undergraduate Texts in Mathematics. Springer (SIE). Indian Reprint 2004.

Additional Reading:

- i. Rosen, Kenneth H. (2012) *Discrete Mathematics and its Applications* (7th ed.). McGraw-Hill Education (India) Pvt. Ltd.

Teaching Plan (GE-2: Discrete Mathematics):

- Week 1:** Compound Statements (and, or, implication, negation, contrapositive, quantifiers), Truth tables.
[2] Chapter 1 (Sections 1.1, and 1.3).
- Week 2:** Basic logical equivalences and its consequences, Logical arguments, Set theory.
[2] Chapter 1 (Sections 1.4, and 1.5), and Chapter 2 (Section 2.1).
- Week 3:** Operation on sets, types of binary relations, Equivalence relations, Congruences and its properties.
[2] Chapter 2 [Sections 2.2, 2.3, and 2.4 (left for convergence)], and Chapter 4 (Section 4.4).
- Week 4:** Partial and total ordering, Lattices.
[2] Chapter 2 (Section 2.5).
- Week 5:** Properties of integers, Division algorithm, Divisibility.
[2] Chapter 4 (Sections 4.1 to 4.1.6).
- Week 6:** Euclidean algorithm, GCD, LCM, Relatively prime.
[2] Chapter 4 (Section 4.2).
- Week 7:** Prime numbers, statement of fundamental theorem of arithmetic, Fermat primes.
[2] Chapter 4 (Sections 4.3 up to 4.3.11, Page 119).
- Week 8:** Mathematical induction, Recursive relations and its solution (characteristics polynomial and generating function).
[2] Chapter 5 (Sections 5.1, 5.3, and 5.4).
- Week 9:** Principles of counting (inclusion /exclusion, pigeon-hole), permutation and combinations (with and without repetition).
[2] Chapter 6 (Section 6.1), Chapter 7 (Sections 7.1 to 7.3).
- Week 10:** Duality principle, lattices as ordered sets.
[1] Sections 1.20, and 2.1 to 2.7.
- Week 11:** Lattices as algebraic structures, Sublattices, Products and Homomorphisms, Distributive lattices.
[1] Chapter 2 (Sections 2.8 to 2.19), Chapter 4 (Sections 4.1 to 4.11)
- Week 12:** Boolean algebras, Boolean polynomials, Minimal forms of Boolean polynomials.
[3] Chapter 1 (Section 2)
- Weeks 13 and 14:** Quinn–McCluskey method, Karnaugh diagrams, Switching circuits and applications of switching circuits.
[3] Chapter 2 (Section 1).

Facilitating the Achievement of Course Learning Outcomes

| Unit No. | Course Learning Outcomes | Teaching and Learning Activity | Assessment Tasks |
|----------|---|--|---|
| 1. | Understand the basic principles of logic, set theory, lattices and Boolean algebra. | (i) Each topic to be explained with examples. (ii) Students to be involved in discussions and encouraged to ask questions. (iii) Students to be given homework/assignments. (iv) Students to be encouraged to give short presentations. | <ul style="list-style-type: none"> • Student presentations. • Participation in discussions. • Assignments and class tests. • Mid-term examinations. • End-term examinations. |
| 2. | Understand the ideas of mathematical induction and basic counting techniques. | | |
| 3. | Proficiently construct logical arguments and rigorous proofs. | | |

Keywords: Truth tables, Set theory, Division algorithm, Fermat primes, Lattices, Boolean polynomials, Switching circuits.

Semester-III Generic Elective (GE) Course -Mathematics

Any *one* of the following:

GE-3: Differential Equations (with Practicals)

GE-3: Linear Programming and Game Theory

GE-3: Differential Equations (with Practicals)

Total Marks: 150 (Theory: 75, Internal Assessment: 25, and Practical: 50)

Workload: 4 Lectures, 4 Practicals (per week) **Credits:** 6 (4+2)

Duration: 14 Weeks (56 Hrs. Theory + 56 Hrs. Practical) **Examination:** 3 Hrs.

Course Objectives: This course includes a variety of methods to solve ordinary and partial differential equations with basic applications to real life problems. It provides a solid foundation to further in mathematics, sciences and engineering through mathematical modeling.

Course Learning Outcomes: The student will be able to:

- i) Solve the exact, linear and Bernoulli equations and find orthogonal trajectories.
- ii) Apply the method of variation of parameters to solve linear differential equations.
- iii) Formulate and solve various types of first and second order partial differential equations.

Unit 1: Ordinary Differential Equations and Applications

First order exact differential equations, Integrating factors and rules to find integrating factors, Linear equations and Bernoulli equations, Orthogonal trajectories and oblique trajectories, Basic theory of higher order linear differential equations, Wronskian and its properties; Solving differential equation by reducing its order.

Unit 2. Explicit Methods of Solving Higher-Order Linear Differential Equations

Linear homogenous equations with constant coefficients, Linear non-homogenous equations, Method of undetermined coefficients, Method of variation of parameters, Cauchy–Euler equations; Simultaneous differential equations.

Unit 3. First and Second Order Partial Differential Equations

Partial differential equations: Basic concepts and definitions. Mathematical problems; First order equations: Classification, Construction, Geometrical interpretation; Method of characteristics, General solutions of first order partial differential equations; Canonical forms and method of separation of variables for first order partial differential equations; Classification of second order partial differential equations; Reduction to canonical forms; Second order partial differential equations with constant coefficients, General solutions.

References:

1. Kreyszig, Erwin. (2011). *Advanced Engineering Mathematics* (10th ed.). Wiley India.
2. Myint-U, Tyn and Debnath, Lokenath (2007). *Linear Partial Differential Equations for Scientist and Engineers* (4th ed.). Birkhäuser Boston. Indian Reprint.
3. Ross, Shepley. L. (1984). *Differential Equations* (3rd ed.). John Wiley & Sons.

Additional reading:

- i. Sneddon I. N. (2006). *Elements of Partial Differential Equations*. Dover Publications.

Practical / Lab work to be performed in a Computer Lab:

Use of Computer Algebra Systems (CAS), for example MATLAB/Mathematica /Maple/Maxima/Scilab etc., for developing the following programs:

1. Solution of first order differential equation.
2. Plotting of second order solution family of differential equation.
3. Plotting of third order solution family of differential equation.
4. Solution of differential equation by variation of parameter method.
5. Solution of system of ordinary differential equations.
6. Solution of Cauchy problem for first order partial differential equations.
7. Plotting the characteristics of the first order partial differential equations.
8. Plot the integral surfaces of first order partial differential equations with initial data.

Teaching Plan (GE-3: Differential Equations):

Weeks 1 and 2: First order ordinary differential equations: Basic concepts and ideas, First order exact differential equation, Integrating factors and rules to find integrating factors.

[3] Chapter 1 (Sections 1.1, and 1.2), and Chapter 2 (Sections 2.1, and 2.2).

[1] Chapter 1 (Sections 1.1, 1.2, and 1.4).

Week 3: Linear equations and Bernoulli equations, Orthogonal trajectories and oblique trajectories.

[3] Chapter 2 (Sections 2.3, and 2.4), and Chapter 3 (Section 3.1).

Weeks 4 and 5: Basic theory of higher order linear differential equations, Wronskian and its properties, Solving a differential equation by reducing its order.

[3] Chapter 4 (Section 4.1).

Weeks 6 and 7: Linear homogenous equations with constant coefficients, Linear non-homogenous equations, Method of undetermined coefficients.

[3] Chapter 4 (Sections 4.2, and 4.3), and

[1] Chapter 2 (Section 2.2).

Weeks 8 and 9: Method of variation of parameters, Cauchy–Euler equations, Simultaneous differential equations.

[3] Chapter 4 (Sections 4.4, and 4.5), and Chapter 7 (Sections 7.1, and 7.3)

Week 10: Partial differential equations: Basic concepts and definitions, Mathematical problems; First order equations: Classification and construction.

[2] Chapter 2 (Sections 2.1 to 2.3).

Weeks 11 and 12: Geometrical interpretation, Method of characteristics, General solutions of first order partial differential equations.

[2] Chapter 2 (Sections 2.4, and 2.5).

Week 13: Canonical forms and method of separation of variables for first order partial differential equations.

[2] Chapter 2 (Sections 2.6, and 2.7)

Week 14: Second order partial differential equations: Classification, Reduction to canonical forms, With constant coefficients, General solutions.

[2] Chapter 4 (Sections 4.1 to 4.4).

Facilitating the Achievement of Course Learning Outcomes

| Unit No. | Course Learning Outcomes | Teaching and Learning Activity | Assessment Tasks |
|----------|---|--|---|
| 1. | Solve the exact, linear and Bernoulli equations and find orthogonal trajectories. | (i) Each topic to be explained with illustrations. | <ul style="list-style-type: none"> • Presentations and class discussions. • Assignments and class tests. • Student presentations. • Mid-term examinations. • Practical and viva-voce examinations. • End-term examinations. |
| 2. | Apply the method of variation of parameters to solve linear differential equations. | (ii) Students to be encouraged to discover the relevant concepts. | |
| 3. | Formulate and solve various types of first and second order partial differential equations. | (iii) Students to be given homework/assignments. (iv) Discuss and solve the theoretical and practical problems in the class. (v) Students to be encouraged to apply concepts to real world problems. | |

Keywords: Integrating factors, Bernoulli equations, Wronskian, Cauchy–Euler equation, First and second order PDE’s.

GE-3: Linear Programming and Game Theory

Total Marks: 100 (Theory: 75 and Internal Assessment: 25)

Workload: 5 Lectures, 1 Tutorial (per week) **Credits:** 6 (5+1)

Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: This course develops the ideas behind the solution of linear programming problem using simplex method, as well as, the solution of transportation and assignment problems. This course also provides an introduction to game theory which makes possible the analysis of the decision making process of two interdependent subjects.

Course Learning Outcomes: This course will enable the students to:

- i) Learn about the simplex method used to find optimal solutions of linear optimization problems subject to certain constraints.
- ii) Write the dual of a linear programming problem.
- iii) Solve the transportation and assignment problems.
- iv) Learn about the solution of rectangular games using graphical method and using the solution of a pair of associated primal-dual linear programming problems.

Unit 1. Linear Programming Problem, Simplex Method and Duality

Graphical method of solution, Basic feasible solutions, Linear programming and convexity; Introduction to the simplex method: Theory of the simplex method, Optimality and unboundedness; Simplex tableau and examples, Artificial variables; Introduction to duality, Formulation of the dual problem with examples and interpretations, Duality theorem.

Unit 2. Transportation and Assignment Problems

Definition and mathematical formulation of transportation problems, Methods of finding initial basic feasible solutions, North West corner rule, Least-cost method, Vogel's approximation method, Algorithm for solving transportation problems; Mathematical formulation and Hungarian method of solving assignment problems.

Unit 3. Two-Person, Zero-Sum Games

Introduction to game theory, Formulation of two-person zero-sum rectangular game, Solution of rectangular games with saddle points, Mixed strategies, Dominance principle, Rectangular games without saddle points, Graphical and linear programming solution of rectangular games.

References:

1. Taha, Hamdy A. (2010). *Operations Research: An Introduction* (9th ed.). Pearson.
2. Thie, Paul R., & Keough, G. E. (2014). *An Introduction to Linear Programming and Game Theory*. (3rd ed.). Wiley India Pvt. Ltd.

Additional Readings:

- i. Hadley, G. (1997). *Linear Programming*. Narosa Publishing House. New Delhi.
- ii. Hillier, F. S., & Lieberman, G. J. (2017). *Introduction to Operations Research* (10th ed.). McGraw-Hill Education (India) Pvt. Ltd.

- iii. Kolman, B. and Hill, D.R. (2003). *Introductory Linear Algebra with Applications*. (7th ed.). Pearson Education. First Indian Reprint 2003.

Teaching Plan (GE-3: Linear Programming and Game Theory):

Week 1: Introduction to linear programming problem: Graphical method of solution, Basic feasible solutions, Linear programming and convexity.

[2] Chapter 2 (Section 2.2), and Chapter 3 (Sections 3.1, 3.2, and 3.9).

Weeks 2 and 3: Introduction to the simplex method: Theory of the simplex method, Optimality and unboundedness.

[2] Chapter 3 (Sections 3.3, and 3.4).

Weeks 4 and 5: Simplex tableau and examples, Artificial variables.

[2] Chapter 3 (Sections 3.5, and 3.6).

Weeks 6 and 7: Introduction to duality, Formulation of the dual problem with examples and interpretations, Statement of the duality theorem with examples.

[2] Chapter 4 (Sections 4.1 to 4.4).

Weeks 8 and 9: Definition and mathematical formulation of transportation problems, Methods of finding initial basic feasible solutions, North West corner rule, Least-cost method, Vogel's approximation method, Algorithm for solving transportation problems.

[1] Chapter 5 (Sections 5.1, and 5.3).

Week 10: Mathematical formulation and Hungarian method of solving assignment problems.

[1] Chapter 5 (Section 5.4).

Weeks 11 and 12: Introduction to game theory, Formulation of two-person zero-sum rectangular game, Solution of rectangular games with saddle points.

[2] Chapter 9 (Sections 9.1 to 9.3).

Weeks 13 and 14: Mixed strategies, Dominance principle, Rectangular games without saddle points, Graphical and linear programming solution of rectangular games.

[2] Chapter 9 (Sections 9.4 to 9.6).

Facilitating the Achievement of Course Learning Outcomes

| Unit No. | Course Learning Outcomes | Teaching and Learning Activity | Assessment Tasks |
|----------|--|---|---|
| 1. | Learn about the simplex method used to find optimal solutions of linear optimization problems subject to certain constraints. Write the dual of a linear programming problem. | (i) Each topic to be explained with examples. (ii) Students to be involved in discussions and encouraged to ask questions. | <ul style="list-style-type: none"> • Student presentations. • Participation in discussions. • Assignments and class tests. • Mid-term examinations. • End-term examinations. |
| 2. | Solve the transportation and assignment problems. | (iii) Students to be given homework/assignments. | |
| 3. | Learn about the solution of rectangular games using graphical method and using the solution of a pair of associated prima-dual linear programming problems. | (iv) Students to be encouraged to give short presentations. | |

Keywords: Basic feasible solutions, Duality, Transportation problems, Assignment problems, Rectangular games, Dominance.

Semester-IV Generic Elective (GE) Course -Mathematics

Any *one* of the following:

GE-4: Numerical Methods (with Practicals)

GE-4: Elements of Analysis

GE-4: Numerical Methods (with Practicals)

Total Marks: 150 (Theory: 75, Internal Assessment: 25, and Practical: 50)

Workload: 4 Lectures, 4 Practicals (per week) **Credits:** 6 (4+2)

Duration: 14 Weeks (56 Hrs. Theory + 56 Hrs. Practical) **Examination:** 3 Hrs.

Course Objectives: The goal of this paper is to acquaint students' various topics in Numerical Analysis such as solutions of nonlinear equations in one variable, interpolation and approximation, numerical differentiation and integration, direct methods for solving linear systems, numerical solution of ordinary differential equations using Computer Algebra System (CAS).

Course Learning Outcomes: After completion of this course, students will be able to:

- i) Find the consequences of finite precision and the inherent limits of numerical methods.
- ii) Appropriate numerical methods to solve algebraic and transcendental equations.
- iii) Solve first order initial value problems of ODE's numerically using Euler methods.

Unit 1: Errors and Roots of Transcendental and Polynomial Equations

Floating point representation and computer arithmetic, Significant digits; Errors: Roundoff error, Local truncation error, Global truncation error; Order of a method, Convergence and terminal conditions; Bisection method, Secant method, Regula-Falsi method, Newton-Raphson method.

Unit 2: Algebraic Linear Systems and Interpolation

Gaussian elimination method (with row pivoting), Gauss-Jordan method; Iterative methods: Jacobi method, Gauss-Seidel method; Interpolation: Lagrange form, Newton form, Finite difference operators, Gregory-Newton forward and backward difference interpolations, Piecewise polynomial interpolation (linear and quadratic).

Unit 3: Numerical Differentiation, Integration and ODE

Numerical differentiation: First and second order derivatives, Richardson extrapolation method; Numerical integration: Trapezoidal rule, Simpson's rule; Ordinary differential equation: Euler's method, Modified Euler's methods (Heun's and midpoint).

References:

1. Chapra, Steven C. (2018). *Applied Numerical Methods with MATLAB for Engineers and Scientists* (4th ed.). McGraw-Hill Education.
2. Fausett, Laurene V. (2009). *Applied Numerical Analysis Using MATLAB*. Pearson. India.
3. Jain, M. K., Iyengar, S. R. K., & Jain R. K. (2012). *Numerical Methods for Scientific and Engineering Computation* (6th ed.). New Age International Publishers. Delhi.

Additional Reading:

- i. Bradie, Brian (2006). *A Friendly Introduction to Numerical Analysis*. Pearson Education India. Dorling Kindersley (India) Pvt. Ltd. Third Impression, 2011.

Practical /Lab work to be performed in the Computer Lab:

Use of Computer Algebra System (CAS), for example MATLAB/Mathematica/Maple/Maxima/Scilab etc., for developing the following Numerical Programs:

1. Bisection method
2. Secant method and Regula–Falsi method
3. Newton–Raphson method
4. Gaussian elimination method and Gauss–Jordan method
5. Jacobi method and Gauss–Seidel method
6. Lagrange interpolation and Newton interpolation
7. Trapezoidal and Simpson’s rule.
8. Euler methods for solving first order initial value problems of ODE’s.

Teaching Plan (Theory of GE-4: Numerical Methods):

Weeks 1 and 2: Floating point representation and computer arithmetic, Significant digits; Errors: Roundoff error, Local truncation error, Global truncation error; Order of a method, Convergence and terminal conditions.

[2] Chapter 1 (Sections 1.2.3, 1.3.1, and 1.3.2).

[3] Chapter 1 (Sections 1.2, and 1.3).

Week 3 and 4: Bisection method, Secant method, Regula–Falsi method, Newton–Raphson method.

[2] Chapter 2 (Sections 2.1 to 2.3).

[3] Chapter 2 (Sections 2.2 and 2.3).

Week 5: Gaussian elimination method (with row pivoting), Gauss–Jordan method;

Iterative methods: Jacobi method, Gauss–Seidel method.

[2] Chapter 3 (Sections 3.1, and 3.2), Chapter 6 (Sections 6.1, and 6.2).

[3] Chapter 3 (Sections 3.2, and 3.4).

Week 6: Interpolation: Lagrange form, and Newton form.

[2] Chapter 8 (Section 8.1). [3] (Section 4.2).

Weeks 7 and 8: Finite difference operators, Gregory–Newton forward and backward difference interpolations.

[3] Chapter 4 (Sections 4.3, and 4.4).

Week 9: Piecewise polynomial interpolation: Linear, and quadratic.

[2] Chapter 8 [Section 8.3 (8.3.1, and 8.3.2)].

[1] Chapter 18 (Sections 18.1 to 18.3).

Weeks 10 and 11: Numerical differentiation: First and second order derivatives, Richardson extrapolation method.

[2] Chapter 11 [Sections 11.1 (11.1.1, 11.1.2 and 11.1.4)]

Weeks 12 and 13: Numerical integration: Trapezoidal rule, Simpson’s rule; Ordinary differential equations: Euler’s method.

[2] Chapter 11 [Section 11.2 (11.2.1, 11.2.2)].

[1] Chapter 22 (Sections 22.1, and 22.2 (up to Page 583)

Weeks 14: Modified Euler’s methods: Heun’s method, The midpoint method.

[1] Chapter 22 (Section 22.3)

Facilitating the Achievement of Course Learning Outcomes

| Unit No. | Course Learning Outcomes | Teaching and Learning Activity | Assessment Tasks |
|----------|--|---|--|
| 1. | Find the consequences of finite precision and the inherent limits of numerical methods. | (i) Each topic to be explained with illustrations. (ii) Students to be encouraged to discover the relevant concepts. | <ul style="list-style-type: none"> • Presentations and class discussions. • Assignments and class tests. |
| 2. | Appropriate numerical methods to solve algebraic and transcendental equations. | (iii) Students to be given homework/assignments. (iv) Discuss and solve the theoretical and practical problems in the class. | <ul style="list-style-type: none"> • Student presentations. • Mid-term examinations. |
| 3. | Solve first order initial value problems of ordinary differential equations numerically using Euler methods. | (v) Students to be encouraged to apply concepts to real world problems. | <ul style="list-style-type: none"> • Practical and viva-voce examinations. • End-term examinations. |

Keywords: Bisection method, Secant method, Regula–Falsi method, Newton–Raphson method, Gauss–Seidel method, Piecewise polynomial interpolation, Richardson extrapolation method, Simpson’s rule.

GE-4: Elements of Analysis

Total Marks: 100 (Theory: 75 and Internal Assessment: 25)

Workload: 5 Lectures, 1 Tutorial (per week) **Credits:** 6 (5+1)

Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: Real analysis provides tools to lay the foundation for further study in subfields, such as calculus, differential equations, and probability. To study this course one needs a background in calculus and a facility with logic and proofs. This course deals with the analytic properties of real numbers, sequences and series, including convergence and limits of sequences of real numbers, the calculus of the real numbers, and convergence of power series.

Course Learning Outcomes: This course will enable the students to:

- i) Understand the real numbers and their basic properties.
- ii) Be familiar with convergent and Cauchy sequences.
- iii) Test the convergence and divergence of infinite series of real numbers.
- iv) Learn about power series expansion of some elementary functions.

Unit 1. Real Numbers and Sequences

Finite and infinite sets, Examples of countable and uncountable sets; Absolute value and the Real line, Bounded sets, Suprema and infima, The completeness property of \mathbb{R} , Archimedean property of \mathbb{R} ; Real sequences, Convergence, sum and product of convergent sequences, Order preservation and squeeze theorem; Monotone sequences and their convergence; Proof of convergence of some simple sequences such as $\frac{(-1)^n}{n}$, $\frac{1}{n^2}$, $\left(1 + \frac{1}{n}\right)^n$, x^n with $|x| < 1$, a_n/n , where a_n is a bounded sequence. Subsequences and the Bolzano–Weierstrass theorem; Limit superior and limit inferior of a bounded sequence; Cauchy sequences, Cauchy convergence criterion for sequences.

Unit 2. Infinite Series of Real Numbers

Definition and a necessary condition for convergence of an infinite series, Geometric series, Cauchy convergence criterion for series; Positive term series, Integral test, Convergence of p -series, Comparison test, Limit comparison test, D’Alembert’s ratio test, Cauchy’s root test; Alternating series, Leibniz test; Absolute and conditional convergence.

Unit 3. Power Series and Elementary Functions

Definition of power series, Radius and interval of convergence, Cauchy–Hadamard theorem, Statement and illustration of term-by-term differentiation, Integration of power series, and Abel’s theorem, Power series expansions for e^x , $\sin x$, $\cos x$, $\log(1 + x)$ and their properties.

References:

1. Bartle, Robert G., & Sherbert, Donald R. (2015). *Introduction to Real Analysis* (4th ed.). Wiley India Edition.
2. Denlinger, Charles G. (2015). *Elements of Real Analysis*. Jones & Bartlett India Pvt. Ltd.
- Ross, Kenneth A. (2013). *Elementary Analysis: The Theory of Calculus* (2nd ed.). Undergraduate Texts in Mathematics, Springer. Indian Reprint.

Additional Reading:

- i. Bilodeau, Gerald G., Thie, Paul R., & Keough, G. E. (2010). *An Introduction to Analysis* (2nd ed.). Jones & Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

Teaching Plan (GE-4: Elements of Analysis):

Weeks 1 and 2: Finite and infinite sets, Examples of countable and uncountable sets; Absolute value of the real line, bounded sets, suprema and infima; Statement of order completeness property of \mathbb{R} , Archimedean property of \mathbb{R} .

[1] Chapter 1 (Section 1.3), and Chapter 2 (Sections 2.2 to 2.4).

Weeks 3 and 4: Real sequences, Convergence, Sum and product of convergent sequences, Order preservation and squeeze theorem.

[1] Chapter 3 (Sections 3.1 and 3.2).

Week 5: Monotone sequences and their convergence, Proof of convergence of some simple sequences such as $\frac{(-1)^n}{n}$, $\frac{1}{n^2}$, $\left(1 + \frac{1}{n}\right)^n$, x^n with $|x| < 1$, a_n/n , where a_n is a bounded sequence.

[1] Chapter 3 (Section 3.3)

Weeks 6 and 7: Subsequences and the Bolzano–Weierstrass theorem (statement and examples), Limit superior and limit inferior of a bounded sequence (definition and examples), Statement and illustrations of Cauchy convergence criterion for sequences.

[1] Chapter 3 (Sections 3.4, and 3.5).

Weeks 8 and 9: Definition and a necessary condition for convergence of an infinite series, Geometric series, Cauchy convergence criterion for series, positive term series, State the integral test and prove the convergence of p -series, Comparison test, Limit comparison test and examples.

[2] Chapter 8 (Section 8.1).

[1] Chapter 3 (Section 3.7).

Week 10: D'Alembert's ratio test, Cauchy's root test.

[2] Chapter 8 (Section 8.2).

Week 11: Alternating series, Leibnitz test; Absolute and conditional convergence.

[2] (Section 8.3).

Week 12: Definition of power series, Radius and interval of convergence, Cauchy–Hadamard theorem.

[3] Chapter 4 [Article 23, 23.1 (without proof)].

[1] Chapter 9 [9.4.7 to 9.4.9 (without proof)].

Week 13: Statement and illustration of term-by-term differentiation, Integration of power series and Abel's theorem.

[3] Chapter 4 (Article 26).

Week 14: Power series expansions for e^x , $\sin x$, $\cos x$, $\log(1 + x)$ and their properties.

[3] Chapter 7 (Article 37).

[1] Chapter 9 (9.4.14).

Facilitating the Achievement of Course Learning Outcomes

| Unit No. | Course Learning Outcomes | Teaching and Learning Activity | Assessment Tasks |
|----------|---|---|---|
| 1. | Understand the real numbers and their basic properties. Be familiar with convergent and Cauchy sequences. | (i) Each topic to be explained with examples. (ii) Students to be involved in discussions and encouraged to ask questions. | <ul style="list-style-type: none">• Student presentations.• Participation in discussions.• Assignments and class tests.• Mid-term examinations.• End-term examinations. |
| 2. | Test the convergence and divergence of infinite series of real numbers. | (iii) Students to be given homework/assignments. | |
| 3. | Learn about power series expansion of some elementary functions. | (iv) Students to be encouraged to give short presentations. | |

Keywords: Countable sets, Completeness property, Bolzano–Weierstrass theorem, Cauchy sequence, Cauchy’s root test, D’Alembert’s ratio test, Cauchy–Hadamard theorem, Abel’s theorem.

Acknowledgments

The following members were actively involved in drafting the LOCF syllabus of Mathematics of Generic Elective Courses for Honours Courses, University of Delhi.

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