International Journal of Astronomy and Astrophysics, 2018, 8, 406-424
http://www.scirp.org/journal/ijaa
ISSN Online: 2161-4725
ISSN Print: 2161-4717

Artificial Equilibrium Points in the Low-Thrust Restricted Three-Body Problem When Both the Primaries Are Oblate Spheroids

Amit Mittal[1], Krishan Pal[2]

[1]Department of Mathematics, A.R.S.D College, University of Delhi, New Delhi, India
[2]Department of Mathematics, Maharaja Agrasen College, University of Delhi, New Delhi, India
Email: to.amitmittal@gmail.com, kpal1987@gmail.com

How to cite this paper: Mittal, A. and Pal, K. (2018) Artificial Equilibrium Points in the Low-Thrust Restricted Three-Body Problem When Both the Primaries Are Oblate Spheroids. *International Journal of Astronomy and Astrophysics*, **8**, 406-424.
https://doi.org/10.4236/ijaa.2018.84028

Received: November 2, 2018
Accepted: December 26, 2018
Published: December 29, 2018

Abstract

This paper studies the existence and stability of the artificial equilibrium points (AEPs) in the low-thrust restricted three-body problem when both the primaries are oblate spheroids. The artificial equilibrium points (AEPs) are generated by canceling the gravitational and centrifugal forces with continuous low-thrust at a non-equilibrium point. Some graphical investigations are shown for the effects of the relative parameters which characterized the locations of the AEPs. Also, the numerical values of AEPs have been calculated. The positions of these AEPs will depend not only also on magnitude and directions of low-thrust acceleration. The linear stability of the AEPs has been investigated. We have determined the stability regions in the *xy*, *xz* and *yz*-planes and studied the effect of oblateness parameters $A_1\left(0<A_1<1\right)$ and $A_2\left(0<A_2<1\right)$ on the motion of the spacecraft. We have found that the stability regions reduce around both the primaries for the increasing values of oblateness of the primaries. Finally, we have plotted the zero velocity curves to determine the possible regions of motion of the spacecraft.

Keywords

Restricted Three-Body Problem, Artificial Equilibrium Points, Low-Thrust, Stability, Oblate Spheroid, Zero Velocity Curves

1. Introduction

Generally, the restricted three-body problem is one of the most important problem in the field of celestial mechanics. In the Restricted Three-Body Problem (R3BP), the mass of the third body (*i.e.*, the spacecraft) is assumed to be

International Journal of Mathematics and Computation,
ISSN 0974-570X (Online), ISSN 0974-5718 (Print),
Vol. 29; Issue No. 1; Year 2018,

Free vibration analysis of two-dimensional non-homogeneous rectangular plates of variable thickness

Renu Saini[1] and Roshan Lal[2]

[1] Department of Applied Science and Humanities,
Inderprastha Engineering College, Ghaziabad;
E-mail: renusaini189@gmail.com

[2]Department of Mathematics,
Indian Institute of Technology Roorkee, Roorkee, India;
E-mail: rlatmfma@iitr.ac.in

ABSTRACT

The free transverse vibrations of two-dimensional non-homogeneous rectangular plates of variable thickness have been analysedbased on Kirchhoff plate theory. The non-homogeneity of the plate material is assumed to vary along both the axial directions of the plate and arise due to the arbitrary variations in the Young's modulus and density of the plate material with the in-plane coordinates. The thickness of the plate is varying bidirectionally and taken as the Cartesian product of linear variations along the two concurrent edges of the plate. The governing differential equation of motion for such a plate model has been derived using Hamilton's energy principle. The resulting equation has been reduced to an eigenvalue problem using two-dimensional generalized differential quadraturemethod for four different boundary conditions. The effect of various parameters such as thickness parameters, non-homogeneity parameters, density parameters, and aspect ratio on the frequencies has been investigated for all the four plates. For a specified plate,three-dimensional mode shapes have been presented. To validate the numerical results comparison has been carried out.

Keywords: Non-homogeneous, variable thickness, isotropic, rectangular,generalized differential quadrature method.

Mathematics Subject Classification:74H45

1. INTRODUCTION

In many engineering applications, the designers have to use variable thickness plates to suit some design requirements, for example, the advantage of material saving, stiffness enhancing weight reduction and hence the cost requirements, or to change the natural frequency of the plate away from the driving frequency. This led to the study of the vibrational characteristics of plates of variable thickness. In particular, rectangular plates are key components in ocean structures and aerospace industry. In this regard, an excellent survey of the work up to 1985 on linear vibration of plates without and with various complicating effects has been presented by Leissa (1985) in his monograph and in a series of review articles (Leissa, 1977; 1978; 1981; 1987). Thereafter, numerous studies dealing with the vibration of rectangular plates with various types of thickness variations such as linear, parabolic, quadratic, exponential, stepped and other arbitrarily varying thickness have appeared in the literature and reported in references (Bhat, 1985; Bhat et al., 1990; Ju et al., 1996; Bert and Malik, 1996; Grigorenko and Tregubenko, 2000; Kang and Kim, 2008; Wang and Wang, 2008; Civalek, 2009; Wilson, 2009; Manna, 2011; Mozafaria and Ayob, 2012; Gupta and Mamta, 2014), to mention a few. In these papers, the thickness variation has been considered unidirectional i.e. the thickness of the plate varies along one direction only. During the survey of literature, the authors have come across a

Renu Saini
7/Mar/2022

A Technique for Fractional Programs with Restriction on Variables

Anuradha Sharma
Department of Mathematics, University of Delhi, India.

Abstract : This research study deals with very special class of single ratio fractional programs.The objective function considered in tjhis study is ratio of non linear functions each separable in nature..Numerator as well as denominator both are transformed to linear function by incorporating/using.the piecewise linear approximation method via grid point and thereby a linear fractional progam is determined which is converted/formulated into a linear program by making use of Charne's and Cooper transformation method. In the course of action, the variables associated being assumed to be bounded i.e. are finite valued. The procedure is explained stepwise by an algorithm and is explained with help of an illustrative example..

Index Terms: Piecewise linear approximation,;Optimization.Grid points search method.;Separability.;Affine Functios,;Linear programming.

I. INTRODUCTION

Consider the separable fractional programming problem

$$\text{(SFPP)} \quad \min \frac{U(x)}{V(x)} = \min \frac{\sum_{j=1}^{n} u_j(x_j) + \alpha}{\sum_{j=1}^{n} v_j(x_j) + \beta}.$$

$$= \min \frac{\sum_{j=1}^{n} p_j x_j^2 + \sum_{j=1}^{n} q_j x_j + \alpha}{\sum_{j=1}^{n} r_j x_j^2 + \sum_{j=1}^{n} s_j x_j + \beta} \qquad (1)$$

subject to $\sum_{j=1}^{n} g_{ij}(x_j) \le b_i \quad \text{for } i = 1, 2, \dots, m$

where $u(x)$ and $v(x)$ are separable non-linear functions of x with the condition that either p_j and r_j are both zero or non-zero; g_{ij} are linear functions of x $(1 \le i \le m,\ 1 \le j \le n)$, $b_i \in \mathbb{R}^n$, $v(x)$ is positive on the constraints set

S = $\{ x_i \in \mathbb{R}^m \ : \ g_{ij}(x_j) \le b_i, 1 \le i \le m, 1 \le j \le n \}$, $x = (x_1, x_2, ..., x_n) \in \mathbb{R}^n$.

S is assumed to be a non-empty convex polyhedron.

Fractional Programminghas been dealt by many authors [1.2.4.5.6.7] Charne's et al. reformulated linear fractional programming problem into a linear programming problem by applying. the transformation. Fractional programs arise in various circumstances/cases: in management science as well as other areas. Maximization of productivity, maximization of return on investment, maximization of cost/time give rise a fractional program.

Nonlinear programming problems have gained great importance since they arie in many fields like financial analysis of firms,selection problem;cutting stock problem; stochastic processing.Single ratio fractional programs generally appeared in the literature in 1960s. Much work has been carried out on theory,classification and applications in this regard. Single ratio fractional programs have been stated in the monographs by Craven[6].A model has been presented by Chang[4] in which auxilliary constraints have been used to linearize the mixed 0–1 fractional programming problem. Here we use approximation technique for finding the solution so as to overcome the complexities/infeasibility of the problem. For solving large scale problems, approximation techniques are employed.The problem presented in this paper is non-concave fractional program.A number of methods of concave programming are available for finding solution by transforming a non-concave fractional program into concave program. Portfolio selection problems and stochastic decision making problems [3.8,9,10] are non-concave fractional programs.

2. METHODOLOGY AND THEORETICAL FRAMEWORK

The following definitions are employed:

Definition 1: "Non-linear programs where the objective functions and the constraint functions can be expressed as the sum of functions, each involving only one variable, are called Separable Programs."

Definition 2:" Let f be a real valued function defined on a convex set S in $\mathbb{R}^m$. The function f is said to be strictly convex on S if

$$f(\lambda x_1 + (1-\lambda)x_2) < \lambda f(x_1) + (1-\lambda) f(x_2) \quad \forall\ x_1, x_2 \in S \text{”}$$

and for each $\lambda \in (0, 1)$.