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Stability of the Artificial Equilibrium Points in the Low-Thrust Restricted Three-Body Problem when the Bigger Primary is a Source of Radiation

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Abstract: This paper investigates the existence and the stability of artificial equilibrium points (AEPs) in the low-thrust restricted three-body problem when the bigger primary is a source of radiation and the smaller one is a point mass. The linear stability of the AEPs has been studied. Firstly, we have derived the equations of motion of the spacecraft in the synodic coordinate system. The AEPs are obtained by cancelling the gravitational and centrifugal forces with continuous control acceleration at the non-equilibrium points. The positions of these AEPs will depend on the magnitude and directions of low-thrust acceleration. Secondly, we have calculated the numerical values of the AEPs and their movement shown graphically for given thrust parameters. We have found the stability regions in the $x-y$, $x-z$, $y-z$-planes and studied the effect of the radiation pressure on the motion of the spacecraft. Further, we have drawn the zero velocity curves (ZVCs) to determine the possible regions of motion in which the spacecraft is free to move.

Keywords: *restricted three-body problem, artificial equilibrium points, low-thrust, stability, radiation pressure, zero velocity curves.*

Mathematics Subject Classification (2010): 70F07, 70F10, 70F15.

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Stability of the Artificial Equilibrium Points in the Low-Thrust Restricted Three-Body Problem when the Smaller Primary is an Oblate Spheroid

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Abstract: The aim of this paper is to study the existence and stability of the artificial equilibrium points (AEPs) in the low-thrust restricted three-body problem when the smaller primary is an oblate spheroid and the bigger one is a point mass. The AEPs are obtained by cancelling the gravitational and centrifugal forces with continuous low-thrust at a non-equilibrium point. The AEPs are calculated numerically and their movement is shown graphically. The positions of these AEPs will depend on the magnitude and directions of the low-thrust acceleration. Firstly, we have linearized the equations of motion of the spacecraft. The linear stability of the AEPs is studied. We have drawn the stability regions in the $x-y$, $x-z$ and $y-z$-planes and studied the effect of the oblateness parameter $A \in (0,1)$ on the motion of the spacecraft. Further, we have determined the zero velocity curves to study the possible boundary regions of motion of the spacecraft. Finally, we have concluded about the effects of the relevant parameters in this problem.

Keywords: *restricted three-body problem; artificial equilibrium points; low-thrust; stability; oblate spheroid; zero velocity curves.*

Mathematics Subject Classification (2010): 70F07, 70F10, 70F15.

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ORIGINAL ARTICLE

The unpredictability of the basins of attraction in photogravitational Chermnykh's problem

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Abstract This paper deals with the photogravitational circular restricted three-body problem, including the effect of circular cluster of material points. The parametric evolution of the position of equilibrium points and the existence of a different number of equilibrium points are illustrated using graphs. A detailed investigation to find out the influence of the radiation pressure q, mass ratio μ and the circular cluster of material points M_b on the geometry of basins of attraction is carried out. Further, the number of iterations required to obtain the desired level of accuracy is recorded and presented using probability distribution diagram. The correlations between the domain of convergence of equilibrium points and the corresponding number of the iterations required to obtain the desired level of accuracy are explained. We monitor the parametric evolution of the basin entropy to reveal the unpredictability in the basins of attractions. It is found that unpredictable (fractal) regions exist in the vicinity of boundaries of the basins of attraction.

Keywords Basin of attraction · Basin entropy · Newton-Raphson method · Fractal · Chermnykh's problem

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1 Introduction

The study of the complex structure of the phase space of nonlinear models in the field of space dynamics and celestial mechanics has gained remarkable attention of those mathematicians who have been working in this field for decades. For a better understanding, several tools have been introduced in the past few years. The basin of attraction (or basin of convergence) is one of the important tools. Our observations took place while following many recent contributions in this field (e.g., Aguirre et al. (2009), Suraj et al. (2018), Zotos (2017), Zotos et al. (2020), Kalvouridis and Gousidou-Koutita (2012), Seoane and Sanjuán (2013), Sprott and Xiong (2015)). In the present work, we have utilised this tool to examine the geometry of the domain of convergence in a specific model.

The Chermnykh's problem is a new kind of restricted three-body problem which was first time studied by Chermnykh (1987). The Chermnykh-like problem has a number of applications in different areas such as celestial mechanics, chemistry, extrasolar planetary system (e.g., Goździewski and Maciejewski (1999), Strand and Reinhardta (1979), Rivera and Lissauer (2000), Jiang and Ip (2001)). The authors have investigated the Chermnykh's problem by including the effect from a circular cluster of material points for planetary systems and found the existence of libration points around the inner part of the circular cluster of material points (see Jiang and Yeh (2003, 2004a,b,c), Abouelmagd et al. (2014) and Kushvah and Kishor (2013)). It has been noticed that the circular cluster of material points has a considerable impact on the structure of the dynamical system (e.g., Jiang and Yeh (2003, 2006), Kushvah (2008), Singh and Taura (2014)). The primaries in a circular restricted three-body problem (CR3BP) are generally considered to be spherical, but in real situations, we ob-

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Magnetic field effect on double-diffusion with magnetic and non-magnetic nanofluids

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ABSTRACT

Magnetic nanofluids also are known as ferrofluids behave differently under magnetic field effect in comparison to non-magnetic nanofluids. The inclusion of magnetic forces term into the governing equations of magnetic media gives rise to the magnetohydrodynamics of magnetic nanofluids, also named as ferrohydrodynamics. This ferrohydrodynaimcs claims promising applications Rosensweig (1985)[1] with the field of new phenomena Berkovsky and Bashtovoy (1996)[2]. In the present numerical study, 2D laminar steady governing equations for double-diffusive natural convection with inclined magnetic forces for magnetic nanofluid Fe_3O_4/H_2O and non-magnetic nanofluid Cu/H_2O inside an exponentially heated and concentrated enclosure have been solved via finite-difference based stream-vorticity approach. Further, a comparative study has been conducted to analyze the impact of the magnetic field upon magnetic nanofluid Fe_3O_4/H_2O and non-magnetic nanofluid Cu/H_2O. Influence of various parameters namely; Rayleigh number (Ra), Lewis number (Le), buoyancy ratio (N), Hartmann number (Ha), nanoparticle volume fraction (ϕ), and magnetic field inclination angle (γ) on heat and mass transfer has been studied in detail. Heat and mass lines visualization techniques have been used for better insight of heat and mass transfer. The present work validates against experimental and numerical benchmark results and found to be in satisfactory acceptance. Total heat and mass transfers strengthen with the rise in inclination angle (γ) of magnetic field. It concludes that the effect of the magnetic field at lower value of Rayleigh number on overall heat transfer utilizing magnetic nanofluid Fe_3O_4/H_2O is more effective than non-magnetic nanofluid Cu/H_2O. Whereas magnetic field effect on mass transfer is more effective at a higher value of Rayleigh number. Percentage reduction in mass transfer is 10% has been found for Fe_3O_4/H_2O, which is less than 12% of Cu/H_2O nanofluid at higher value of volume concentration of nanoparticle.

1. Introduction

Magnetic nanofluids or ferrofluids are the different and special category of nanofluids. In comparison to non-magnetic nanofluids which are colloids of nonmagnetic nanoparticle, such as Cu, Al_2O_3 etc. in base fluids, ferrofluids behave differently under controllable moderate magnetic field [3]. Magnetic nanofluids are the stable suspension of magnetic nanoparticles, such as Fe or Fe-C, Co, $CoFe_2O_4$, $\gamma - Fe_3O_4$, Fe_2O_3 in a base fluid [4]. In double-diffusive convection, heat and mass transfer is the combined effect of temperature and concentration gradients. With comparison to non-magnetic fluids, thermal gradients in magnetic suspension nanofluids can render the magnetic force via the temperature-dependent magnetic susceptibility [5]. Mass transfer in magnetic media is due to multivariate transfer phenomena of direct concentration and magnetic diffusion and also due to cross thermal diffusion [6]. Double-diffusive natural convection is widely used in many engineering fields, e.g., geophysics, solar collector, food engineering, bioengineering, distillation, drying technologies, building engineering, and geothermal energy recovery are described in the literature [7–11]. Convection under magnetic field has numerous applications in engineering problems such as crystal growth [12,13], cooling electronic devices [14], and biological systems [15]. In medical science, water-based magnetic fluids are used for immobilization of biomolecules and drug delivery [16].

Mebarek-Oudina and his group members have been done some interesting work on the enhancement of heat transfer during convection process utilizing nanofluids in the absent magnetic field effect [17] as well as under the influence of magnetic field [18–22]. Numerous experimental and numerical studies have been done on double-diffusive convection in different types of geometries using convectional fluids (water, air, ethylene glycol, etc.) with or without magnetic field effect. Few of

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Periodic orbits in the restricted problem of three bodies in a three-dimensional coordinate system when the smaller primary is a triaxial rigid body

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Abstract

In this paper, we have studied the equations of motion for the problem, which are regularised in the neighbourhood of one of the finite masses and the existence of periodic orbits in a three-dimensional coordinate system when $\mu = 0$. Finally, it establishes the canonical set (l, L, g, G, h, H) and forms the basic general perturbation theory for the problem.

Keywords: restricted three problem, Levi-Civita transformation, periodic orbits, triaxial rigid body
AMS 2010 codes: 70F15

1 Introduction

In this paper, we wish to study the three-dimensional generalisation of the problem studied by Bhatnagar (12–14) for the circular case. Since the Hamilton-Jacobi equation for generating a solution takes an unmanageable form for any solution, we have assumed that the third coordinate ζ_3 of the infinitesimal mass is of the $0(\mu)$. It will be interesting to observe that various equations and results worked out by Bhatnagar can be deduced from our results. In Section 2 we have determined the canonical form of the equations of motion, and in Section 3 these equations are regularised by the generalised Levi-Civita's transformation for three dimensions. Eqs (20)–(22) establish the canonical set (l, L, g, G, h, H) and Eq (32) form the basis of the general perturbation theory for the problem under consideration. During the last few years, many mathematician and astronomers have studied different types of periodic orbits in the restricted problem. Some of them are Giacaglia (7), Mayer and Schmidt (17), Markellos (19), Hadjidemetriou (10,11), Bhatnagar and Taqvi (15), Gomez and Noguera (8), Kadrnoska and Hadrava (9), Peridios et al. (21), Ahmad (1), Elipe and Lara (4), Mathlouthi (23), Scuflaire (22), Caranicolas (20), Poddar et al. (5, 6), Abouelmagd and Guirao (2) and Abouelmagd et al. (3). In this work, we have presented an analytical study of the existence of periodic orbits for $\mu = 0$ in the restricted problem of three

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